Gauge-independent Thermal β Function in Yang-Mills Theory via the Pinch Technique

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Abstract

It is proposed to use the pinch technique to obtain the gauge-independent thermal β function β_T in a hot Yang-Mills gas. Calculations of β_T are performed at one-loop level in four different gauges, (i) the background field method with an arbitrary gauge parameter, (ii) the Feynman gauge, (iii) the Coulomb gauge, and (iv) the temporal axial gauge. When the pinch contributions to the gluon self-energy are included, the same result is derived for β_T in all four cases.

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1 Introduction

It is important for the study of the quark-gluon plasma and/or the evolution of the early Universe to fully understand the behaviour of the effective coupling constant $\alpha_s(= g^2/4\pi)$ in QCD at high temperature. The running of α_s with the temperature T and the external momentum κ is governed by the thermal β function β_T [1]. However, the previous calculations of β_T have exposed various problems: (i) strong vertex dependence—the coupling strongly depends on which vertex is chosen to renormalize α_s [2]-[4]; (ii) severe dependence on the vertex-momentum configuration even after a vertex is specified [5]; (iii) the gauge-fixing dependence [2]. To circumvent these difficulties, it was then proposed to use [6] the Vilkovisky-DeWitt effective action [7][8] or to use [9]-[11] the background field method (BFM) for the calculation of β_T at one-loop. (In Yang-Mills theories the Vilkovisky-DeWitt effective action formalism coincides with BFM in the background Landau gauge [8].)

First introduced by DeWitt [12], BFM is a technique for quantizing gauge field theories while retaining explicit gauge invariance for the background fields. Since the Green's functions constructed by BFM manifestly maintain gauge invariance, they obey the naive QED-like Ward-Takahashi identities and the renormalized gauge coupling is defined only through the gluon wave-function renormalization constant. As a result, when the static and symmetric point is chosen for the renormalization condition of the three-gluon vertex, β_T is obtained in BFM from [6][9]-[11]

$$\beta_T \equiv T\frac{dg(T,\kappa)}{dT} = \frac{g}{2\kappa^2}T\frac{d\Pi_T(T,\kappa)}{dT}. \tag{1.1}$$

where $\Pi_T(T,\kappa) = \Pi_T(T, k_0 = 0, \kappa = |\vec{\mathbf{k}}|)$ is the transverse function of the gluon self-energy $\Pi_{\mu\nu}$ at the static limit. Due to the $O(3)$ invariance, the spatial part of the gluon self-energy Π_{ij} is expressed as follows:

$$\Pi_{ij}(k) = \Pi_T(\delta_{ij} - \frac{k_i k_j}{\vec{\mathbf{k}}^2}) + \Pi_L\frac{k_i k_j}{\vec{\mathbf{k}}^2} \tag{1.2}$$

and Π_T can be extracted by applying the projection operator

$$t_{ij} = \frac{1}{2}(\delta_{ij} - \frac{k_i k_j}{\vec{\mathbf{k}}^2}) \tag{1.3}$$

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to Π_{ij}.

The thermal β function has been calculated in BFM at one-loop level for the cases of the gauge parameter $\xi_Q = 0$ [6][10], $\xi_Q = 1$ [9] and $\xi_Q =$ an arbitrary number [11]. The results are expressed in a form,

$$\beta_T^{BFM} = \frac{g^3 N}{2} \left\{ \frac{7}{16} - \frac{1}{8}(1 - \xi_Q) + \frac{1}{64}(1 - \xi_Q)^2 \right\} \frac{T}{\kappa}, \qquad (1.4)$$

where N is the number of colors. Contrary to the case of the QCD β function at zero temperature, β_T^{BFM} is dependent on the gauge-parameter ξ_Q. The reason why we have obtained ξ_Q-dependent β_T in BFM is that the contributions to β_T come from the finite part of the gluon self-energy $\Pi_{\mu\nu}$ and that BFM gives ξ_Q-dependent finite part for $\Pi_{\mu\nu}$. The latter notion had already been known [7][8] but was recently brought to light again in a different context [13][14].

The purpose of the present paper is to propose the use of the pinch technique (PT) to obtain the gauge-independent β_T in a hot Yang-Mills gas. The preliminary results of this paper were given in Ref.[15]. The PT was proposed some time ago by Cornwall [16] for an algorithm to form new gauge-independent proper vertices and new propagators with gauge-independent self-energies. First it was used to obtain the one-loop gauge-independent effective gluon self-energy and vertices in QCD [17] and then it has been applied to the standard model [18]. The application of PT to QCD at high temperature was first made by Alexanian and Nair [19] to calculate the gap equation for the magnetic mass to one-loop order.

In the framework of PT, the one-loop gluon self-energy, when the one-loop pinch contributions from the vertex and box diagrams are added, becomes gauge-independent. In this way we can construct the gauge-independent effective gluon self-energy. As in the case of BFM, the effective gluon self-energy constructed by PT obey the naive QED-like Ward-Takahashi identity. Thus we can use the same Eq.(1.1) to calculate β_T in the framework of PT. More importantly, PT gives the gauge-independent results *up to the finite terms*, since they are constructed from S-matrix. It was shown recently [20] that BFM with the gauge parameter $\xi_Q = 1$ reproduces the PT results at one-loop order. However, for $\xi_Q \neq 1$, this coincidence does not hold any more. In fact, BFM gives at one-loop order the gluon self-energy whose finite part is ξ_Q-dependent. Interestingly enough, Papavassiliou [13] showed

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that when PT is applied to BFM for $\xi_Q \neq 1$ to construct the effective gluon self-energy, the gauge-parameter dependence of the finite part disappears and the previous $\xi_Q = 1$ result (or the universal PT result) is recovered. To author's knowledge, there exists, so far, only one approach, i.e. PT, which gives the gauge-independent gluon self-energy *including the finite terms*. And indeed these finite terms give contributions to β_T. This notion inspires the use of PT for the calculations of β_T.

The paper is organized as follows. In the next section, we develop the general prescription necessary for extracting the pinch contributions to the gluon self-energy from the one-loop quark-quark scattering amplitude. Using this prescription, in Sect.3, we give the *complete* expressions of the one-loop pinch contributions calculated in four different gauges, (i) the BFM with an arbitrary gauge parameter, (ii) the Feynman gauge, (iii) the Coulomb gauge, and (iv) the temporal axial gauge. (The expressions given in Ref.[15] were enough for the calculation of β_T but not in a complete form.) Then we show in detail that when two contributions to β_T are added, one from the ordinary one-loop gluon self-energy and the other from "pinch" counterpart, we obtain the same β_T in the above four different gauges. Sect.4 is devoted to the conclusions and discussions. In addition, we present two Appendices. In Appendix A, we give one-loop pinch contributions to the gluon self-energy from the vertex diagrams of the first kind, of the second kind and box diagrams, separately, in the above four different gauges. In Appendix B, we list the formulas for thermal one-loop integrals necessary for calculating β_T in this paper.

2 Pinch technique

In this section we explain how to obtain the one-loop pinch contributions to the gluon self-energy. Let us consider the S-matrix element T for the elastic quark-quark scattering at one-loop order in the Minkowski space, assuming that quarks have the same mass m. Besides the self-energy diagram in Fig.1, the vertex diagrams of the first kind and the second kind, and the box diagrams, which are shown in Fig.2(a), Fig.3(a), and Fig.4(a), respectively, contribute to T. Such contributions are, in general, gauge-dependent while the sum is gauge-independent. Then we

single out the "pinch parts" of the vertex and box diagrams, which are depicted in Fig.2(b), Fig.3(b), and Fig.4(b). They emerge when a γ^μ matrix on the quark line is contracted with a four-momentum k_μ offered by a gluon propagator or a bare three-gluon vertex. Such a term triggers an elementary Ward identity of the form

$$\slashed{k} = (\slashed{p} + \slashed{k} - m) - (\slashed{p} - m). \tag{2.1}$$

The first term removes (pinches out) the internal quark propagator, whereas the second term vanishes on shell, or *vice versa* . This procedure leads to contributions to T with one or two less quark propagators and, hence, let us call these contributions as T_P, "pinch parts" of T.

Next we extract from T_P the pinch contributions to the gluon self-energy $\Pi^{\mu\nu}$. First note that the contribution of the gluon self-energy diagram to T is written in the form

$$T^{(S.E)} = [T^a \gamma^\alpha] D_{\alpha\mu}(k) \Pi^{\mu\nu} D^{\nu\beta}(k)[T^a \gamma^\beta], \tag{2.2}$$

where $D(k)$ is a gluon propagator, T^a is a representation matrix of $SU(N)$, and γ^α and γ^β are γ matrices on the external quark lines. The pinch contribution $\Pi^{\mu\nu}_P$ to T_P should have the same form. Thus we must take away $[T^a \gamma^\alpha] D_{\alpha\mu}(k)$ and $D^{\nu\beta}(k)[T^a \gamma^\beta]$ from T_P. For that purpose we use the following identity satisfied by the gluon propagator and its inverse:

$$
\begin{aligned}
g_\alpha^\beta &= D_{\alpha\mu}(k)[D^{-1}]^{\mu\beta}(k) = D_{\alpha\mu}(k)[-k^2 d^{\mu\beta}] + k_\alpha \text{ term} \\
&= D_{\alpha\mu}^{-1}(k) D^{\mu\beta}(k) = [-k^2 d_{\alpha\mu}] D^{\mu\beta}(k) + k_\beta \text{ term},
\end{aligned} \tag{2.3}
$$

where

$$d^{\mu\nu} = g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}. \tag{2.4}$$

The k_α and k_β terms give null results when they are contracted with γ_α and of γ_β , respectively, of the external quark lines.

The pinch part of the one-loop vertex diagrams of the first kind depicted in Fig.2(b) plus their mirror graphs has a form

$$T_P^{(V_1)} = \mathcal{A}[T^a \gamma^\alpha] D_{\alpha\beta}(k)[T^a \gamma^\beta] , \tag{2.5}$$

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where $\mathcal{A}$ (also $\mathcal{B}_0$, $\mathcal{B}_{ij}$, $\mathcal{C}_0$, and $\mathcal{C}_{ij}$ in the equations below) contains a loop integral. Using Eq.(2.3) we find

$$\gamma^\alpha D_{\alpha\beta}(k)\gamma^\beta = \gamma^\alpha D_{\alpha\mu}(k)[-k^2 d^{\mu\nu}]D_{\nu\beta}(k)\gamma^\beta \tag{2.6}$$

Thus the contributions to $\Pi^{\mu\nu}$ from the vertex diagrams of the first kind are written as

$$\Pi_P^{\mu\nu(V_1)} = [-k^2 d^{\mu\nu}]\mathcal{A} \tag{2.7}$$

The pinch part of the one-loop vertex diagrams of the second kind depicted in Fig.3(b) plus their mirror graphs has a form

$$T_P^{(V_2)} = \left[T^a \left\{ [\gamma^\kappa]\mathcal{B}_0 + \sum_{i,j} [\slashed{p}_i]p_j^\kappa \mathcal{B}_{ij} \right\} \right] D_{\kappa\beta}(k)[T^a\gamma^\beta]$$
$$+(\mu \leftrightarrow \nu) , \tag{2.8}$$

where $(\mu \leftrightarrow \nu)$ terms are the contributions from mirror diagrams, and p_i and p_j are four-momenta appearing in the diagrams. Using Eq.(2.6) and

$$[\slashed{p}_i]p_j^\kappa D_{\kappa\beta}(k) = [\gamma^\alpha]D_{\alpha\mu}(k)[-k^2 d^{\mu\lambda}]p_{i\lambda}p_j^\nu D_{\nu\beta}(k) , \tag{2.9}$$

we obtain for the contributions to $\Pi^{\mu\nu}$ from the vertex diagram of the second kind

$$\Pi_P^{\mu\nu(V_2)} = [-k^2 d^{\mu\nu}]\mathcal{B}_0 + [-k^2 d^{\mu\lambda}]\sum_{i,j} p_{i\lambda}p_j^\nu \mathcal{B}_{ij}$$
$$+(\mu \leftrightarrow \nu) . \tag{2.10}$$

The pinch part of the one-loop box diagrams depicted in Fig.4(b) has a form

$$T_P^{(Box)} = [T^a]\left\{ [\gamma^\alpha][\gamma_\alpha]\mathcal{C}_0 + \sum_{i,j} [\slashed{p}_i][\slashed{p}_j]\mathcal{C}_{ij} \right\}[T^a]. \tag{2.11}$$

Again from Eq.(2.3) we see that $[\gamma^\alpha][\gamma_\alpha]$ and $[\slashed{p}_i][\slashed{p}_j]$ are rewritten as

$$[\gamma^\alpha][\gamma_\alpha] = [\gamma^\alpha]D_{\alpha\mu}(k)[k^4 d^{\mu\nu}]D_{\nu\beta}(k)[\gamma_\beta] \tag{2.12}$$

$$[\slashed{p}_i][\slashed{p}_j] = [\gamma^\alpha]D_{\alpha\mu}(k)[k^4 d^{\mu\lambda}d^{\nu\tau}p_{i\lambda}p_{j\tau}]D_{\nu\beta}(k)[\gamma_\beta] \tag{2.13}$$

and thus we obtain for the contributions to $\Pi^{\mu\nu}$ from the box diagrams

$$\Pi_P^{\mu\nu(Box)} = [k^4 d^{\mu\nu}]\mathcal{C}_0 + [k^4 d^{\mu\lambda}d^{\nu\tau}]\sum_{i,j} p_{i\lambda}p_{j\tau}\mathcal{C}_{ij} . \tag{2.14}$$

It is observed that the prescription developed here is general and can be applied to the calculation of the one-loop pinch contributions in any gauge.

3   Calculation of Thermal β Function

In this section it will be shown that we obtain the same β_T in the framework of PT even when we calculate in four different gauges, (i) the background field method with an arbitrary gauge parameter, (ii) the Feynman gauge, (iii) the Coulomb gauge, and (iv) the temporal axial gauge. From now on we use the imaginary time formalism of thermal field theory. Thus the loop integral in the Minkowski space is replaced with the following one:

$$- i \int \frac{d^4 p}{(2\pi)^4} \Rightarrow \int dp \equiv \int \frac{d^3 p}{8\pi^3} T \sum_n, \tag{3.1}$$

where the summation goes over the integer n in $p_0 = 2\pi i n T$.

3.1   The Background Field Method

In the background field method (BFM) with an arbitrary gauge parameter ξ_Q, the gluon propagator $i D^{\mu\nu}_{ab(BFM)} = i\delta_{ab} D^{\mu\nu}_{(BFM)}$ and the three-gluon vertex $\tilde{\Gamma}^{abc}_{\lambda\mu\nu}$ with one background gluon field A^b_μ are given as follows [21]:

$$D^{\mu\nu}_{(BFM)} = -\frac{1}{k^2}\left[g^{\mu\nu} - (1 - \xi_Q)\frac{k^\mu k^\nu}{k^2}\right], \tag{3.2}$$

and

$$\tilde{\Gamma}^{abc}_{\lambda\mu\nu}(p, k, q) = g f^{abc}\left[(1 - \frac{1}{\xi_Q})\Gamma^P_{\lambda\mu\nu}(p, k, q) + \Gamma^F_{\lambda\mu\nu}(p, k, q)\right], \tag{3.3}$$

where

$$\begin{aligned}
\Gamma^P_{\lambda\mu\nu}(p, k, q) &= p_\lambda g_{\mu\nu} - q_\nu g_{\lambda\mu} \\
\Gamma^F_{\lambda\mu\nu}(p, k, q) &= 2k_\lambda g_{\mu\nu} - 2k_\nu g_{\lambda\mu} - (2p + k)_\mu g_{\lambda\nu},
\end{aligned} \tag{3.4}$$

and f^{bac} is the structure constant of the group $SU(N)$. In the vertex, k_μ is taken to be the momentum of the background field and each momentum flows inward and, thus, $p + k + q = 0$.

The gluon self-energy at one-loop level was calculated in BFM with an arbitrary gauge parameter ξ_Q [22] and is given as follows:

$$\Pi^{\mu\nu}_{(BFM)}(k) = Ng^2 \int dp \left[\frac{1}{p^2 q^2}(4p^\mu p^\nu - 2p^2 g^{\mu\nu} - k^\mu k^\nu + 4k^2 d^{\mu\nu})\right.$$

$$-(1-\xi_Q)\frac{k^2}{p^4q^2}\left\{(k^2-q^2)d^{\mu\nu}+\left[(d^{\mu\alpha}p_\alpha p^\nu+d^{\nu\beta}p_\beta p^\mu)+(p\leftrightarrow q)\right]\right\}$$

$$+\frac{1}{2}(1-\xi_Q)^2\frac{k^4}{p^4q^4}d^{\mu\alpha}d^{\nu\beta}p_\alpha p_\beta\Bigg],\tag{3.5}$$

where it is understood that the loop variables are related by $k+p+q=0$.

The transverse function in the static limit, $\Pi_T^{(BFG)}(k_0=0,\kappa=|\vec{\mathbf{k}}|)$, can be extracted by applying the projection operator t_{ij} to $\Pi_{(BFM)}^{ij}(k)$ and we have

$$
\begin{aligned}
\Pi_T^{(BFM)}(T,\kappa) &= 2Ng^2\int dp\frac{1}{p^2q^2}\left[p_0^2+2\vec{\mathbf{k}}^2-\frac{(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2}{\vec{\mathbf{k}}^2}\right]\\
&\quad+Ng^2(1-\xi_Q)\int dp\left\{\frac{\vec{\mathbf{k}}^2}{p^2q^2}+\frac{2}{p^4q^2}\left[-\vec{\mathbf{k}}^2(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})+\vec{\mathbf{k}}^2\vec{\mathbf{p}}^2-(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2\right]\right\}\\
&\quad+\frac{N}{4}g^2(1-\xi_Q)^2\vec{\mathbf{k}}^2\int dp\frac{\vec{\mathbf{k}}^2\vec{\mathbf{p}}^2-(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2}{p^4q^4}.
\end{aligned}\tag{3.6}
$$

After the p_0 summation and the angular integration we obtain

$$
\begin{aligned}
\Pi_T^{(BFM)}(T,\kappa) &= \frac{Ng^2}{4\pi^2}\int_0^\infty dp\,p\,n(p)\left[-2+\left(\frac{2p}{\kappa}+\frac{7\kappa}{2p}\right)\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\right]\\
&\quad-Ng^2(1-\xi_Q)\frac{\kappa}{4\pi^2}\int_0^\infty dp\,p\,n(p)\left\{-\frac{4p\kappa}{(2p+\kappa)(2p-\kappa)}+\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\right\}\\
&\quad+\frac{N}{4}g^2(1-\xi_Q)^2\\
&\qquad\times\frac{\kappa}{4\pi^2}\int_0^\infty dp\,p\,n(p)\left\{-\frac{2\kappa^2}{(2p+\kappa)(2p-\kappa)}+\frac{\kappa}{2p}\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\right\}
\end{aligned}
$$
$$\tag{3.7}$$

where, in the r.h.s., $p\equiv|\vec{\mathbf{p}}|$, distribution function, and we have used formulas given in Appendix B for the thermal one-loop integrals. Also listed in Appendix B are the formulas useful for the $\kappa<<T$ expansion. In the limit $\kappa<<T$ we get

$$\Pi_T^{(BFM)}(T,\kappa)\approx Ng^2\kappa T\left\{\frac{7}{16}-\frac{1}{8}(1-\xi_Q)+\frac{1}{64}(1-\xi_Q)^2\right\}+\mathcal{O}(\kappa^2).\tag{3.8}$$

Using this expression for Π_T in Eq.(1.1), Elmfors and Kobes obtained Eq.(1.4) for β_T^{BFM} which is indeed gauge-parameter ξ_Q dependent [11].

Now we evaluate the pinch contributions to Π_T. We consider the quark-quark scattering at one-loop order, using the gluon propagator and the three-gluon vertex

given in Eqs.(3.2)-(3.4). We pinch out the internal quark propagators and obtain the pinch parts of the scattering amplitude T_P. Since the inverse of the propagator $D^{\mu\nu}_{(BFM)}(k)$ is given by

$$[D^{-1}_{(BFM)}]^{\mu\nu}(k) = -k^2\left[g^{\mu\nu} - (1 - \frac{1}{\xi_Q})\frac{k^\mu k^\nu}{k^2}\right], \tag{3.9}$$

$D_{(BFM)}$ and $D^{-1}_{(BFM)}$ satisfy the relations in Eq.(2.3)

$$D^{(BFM)}_{\alpha\mu}(k)[D^{-1}_{(BFM)}]^{\mu\beta}(k) = D^{(BFM)}_{\alpha\mu}(k)[-k^2 d^{\mu\beta}] + \frac{k_\alpha k^\beta}{k^2}$$

$$D^{-1}_{(BFM)\alpha\mu}(k)D^{\mu\beta}_{(BFM)}(k) = [-k^2 d_{\alpha\mu}]D^{\mu\beta}_{(BFM)}(k) + \frac{k_\alpha k^\beta}{k^2}. \tag{3.10}$$

Thus we can follow the prescription explained in Sect.2 and extract the pinch contributions to the gluon self-energy from T_P. The sum is expressed as

$$\Pi^{\mu\nu}_{P(BFM)}(k) = Ng^2\Big[(1-\xi_Q)k^2 d^{\mu\nu}\int dp\frac{-2kp}{p^4 q^2}$$

$$+\frac{1}{2}(1-\xi_Q)^2 k^4 d^{\mu\alpha}d^{\nu\beta}\int dp\frac{-p_\alpha p_\beta}{p^4 q^4}\Big], \tag{3.11}$$

which was first obtained by Papavassiliou [13]. For completeness the one-loop pinch contributions in BFM from the vertex diagrams of the first kind [Fig.2(b) and its mirror graph], the vertex diagrams of the second kind [Fig.3(b) and its mirror graph] and the box-diagrams [Fig.4(b)] are separately given in Appendix A.

Applying the projection operator t_{ij} to the spatial part of $\Pi^{\mu\nu}_{P(BFG)}(k_0 = 0, \kappa = |\vec{k}|)$, we obtain

$$\Pi^{P(BFM)}_T(T, \kappa) = Ng^2(1-\xi_Q)\kappa^2\int dp\frac{2\vec{k}\cdot\vec{p}}{p^4 q^2}$$

$$-\frac{N}{4}g^2(1-\xi_Q)^2\kappa^2\int dp\frac{\vec{k}^2\vec{p}^2 - (\vec{k}\cdot\vec{p})^2}{p^4 q^4}. \tag{3.12}$$

Clearly, the $(1-\xi_Q)^2$ terms of $\Pi^{(BFM)}_T$ in Eq.(3.6) and of $\Pi^{P(BFM)}_T$ in Eq.(3.12) are the same but have an opposite sign, and so when they are combined they are cancelled out. Also we can see the $(1-\xi_Q)$ terms of $\Pi^{(BFM)}_T$ and of $\Pi^{P(BFM)}_T$ cancel when combined, due to the identity

$$\int dp\frac{\vec{k}^2\vec{p}^2 - (\vec{k}\cdot\vec{p})^2}{p^4 q^2} = -\frac{1}{2}\vec{k}^2\int dp\frac{1}{p^2 q^2}. \tag{3.13}$$

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Thus we find the sum

$$
\begin{aligned}
\Pi_T(T,\kappa) \quad &\equiv \Pi_T^{(BFM)}(T,\kappa) + \Pi_T^{P(BFM)}(T,\kappa) \\
&= \frac{Ng^2}{4\pi^2} \int_0^\infty dp\, p\, n(p) \left[-2 + \left(\frac{2p}{\kappa} + \frac{7\kappa}{2p} \right) \ln\left| \frac{2p+\kappa}{2p-\kappa} \right| \right]
\end{aligned}
\qquad (3.14)
$$

is gauge-parameter ξ_Q independent. In the limit $\kappa << T$

$$
\Pi_T(T,\kappa) \approx Ng^2 \kappa T \frac{7}{16} + \mathcal{O}(\kappa^2)
\qquad (3.15)
$$

and this gives a ξ_Q-independent thermal β function

$$
\beta_T = g^3 N \frac{7}{32} \frac{T}{\kappa}.
\qquad (3.16)
$$

Actually we will see below that the expression is independent of the choice of gauge-fixing. Note that the result coincides with β_T^{BFM} in Eq.(1.4) with $\xi_Q = 1$ [9].

It should be remarked that not only the sum of $\Pi_T^{(BFM)}$ and $\Pi_T^{P(BFM)}$, but also the sum of the one-loop gluon self-energy $\Pi_{(BFM)}^{\mu\nu}(k)$ and the corresponding pinch contribution $\Pi_{P(BFM)}^{\mu\nu}(k)$ becomes ξ_Q-independent [13]. Indeed the $(1 - \xi_Q)$ term of $\Pi_{(BFM)}^{\mu\nu}(k)$ in Eq.(3.5) can be rewritten as

$$
Ng^2(1-\xi_Q)k^2 d^{\mu\nu} \int dp \frac{2kp}{p^4 q^2}
\qquad (3.17)
$$

with the help of the identity

$$
\int dp \frac{4p_\mu p_\nu}{p^4 q^2} = \int dp \frac{g_{\mu\nu}}{p^2 q^2} - \int dp \frac{2k_\mu p_\nu}{p^4 q^2},
\qquad (3.18)
$$

where $p + q + k = 0$ is understood and

$$
d_{\mu\alpha} \int dp \frac{p^\alpha}{p^4 q^2} = 0.
\qquad (3.19)
$$

Thus both the $(1 - \xi_Q)$ and $(1 - \xi_Q)^2$ terms of $\Pi_{(BFM)}^{\mu\nu}(k)$ in Eq.(3.5) cancel out with the corresponding ones in $\Pi_{P(BFM)}^{\mu\nu}(k)$ in Eq.(3.11) and there remains the ξ_Q independent (actually it is independent of the choice of gauge-fixing [23]) expression for the effective gluon self-energy,

$$
\widehat{\Pi}^{\mu\nu}(k) = Ng^2 \int dp \frac{1}{p^2 q^2} (4p^\mu p^\nu - 2p^2 g^{\mu\nu} - k^\mu k^\nu + 4k^2 d^{\mu\nu}).
\qquad (3.20)
$$

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Of course we can get the result of Eq.(3.14) directly by applying t_{ij} to the above $\widehat{\Pi}^{\mu\nu}(k)$ and by integration.

It is amusing to note that in BFM with the choice of $\xi_Q = 1$ there is no pinch contribution to the gluon self-energy at one-loop order. This is due to the fact with $\xi_Q = 1$ the longitudinal term $k_\mu k_\nu$ of the gauge boson propagator disappears and the three-gluon vertex $\widetilde{\Gamma}^{abc}_{\lambda\mu\nu}$ is made up of only $\Gamma^F_{\lambda\mu\nu}$ and, hence, there appears no four-momentum which triggers the Ward identity of Eq.(2.1) and pinches out a quark propagator. Therefore when we work with the choice of $\xi_Q = 1$ in BFM and calculate the gluon self-energy diagrams, we directly obtain the the gauge-independent expression $\widehat{\Pi}^{\mu\nu}(k)$ in Eq.(3.20) [20].

3.2 *The Feynman Gauge*

In the Feynman gauge (FG) (the covariant gauge with $\xi = 1$) the gluon propagator, $iD^{\mu\nu}_{ab(FG)} = i\delta_{ab}D^{\mu\nu}_{(FG)}$, has a very simple form

$$D^{\mu\nu}_{(FG)}(k) = \frac{-1}{k^2}g^{\mu\nu}, \tag{3.21}$$

and the three-gluon vertex is expressed as

$$\Gamma^{abc}_{\lambda\mu\nu}(p,k,q) = gf^{bac}\left[\Gamma^P_{\lambda\mu\nu}(p,k,q) + \Gamma^F_{\lambda\mu\nu}(p,k,q)\right] \tag{3.22}$$

where $\Gamma^P_{\lambda\mu\nu}(p,k,q)$ and $\Gamma^F_{\lambda\mu\nu}(p,k,q)$ are given in Eq.(3.4). The expression of the one-loop gluon self-energy in FG is well known:

$$\Pi^{\mu\nu}_{(FG)}(k) = Ng^2\int dp\frac{1}{p^2q^2}\left[4p^\mu p^\nu - 2p^2 g^{\mu\nu} - k^\mu k^\nu + 2k^2 d^{\mu\nu}\right]. \tag{3.23}$$

Since the inverse of the gluon propagator is given by

$$[D^{-1}_{(FG)}]^{\mu\nu}(k) = -k^2 g^{\mu\nu}, \tag{3.24}$$

$D_{(FG)}$ and its inverse satisfy

$$\begin{aligned}
D^{(FG)}_{\alpha\mu}(k)[D^{-1}_{(FG)}]^{\mu\beta}(k) &= D^{(FG)}_{\alpha\mu}(k)[-k^2 d^{\mu\beta}] + \frac{k_\alpha k^\beta}{k^2} \\
D^{-1}_{(FG)\alpha\mu}(k)D^{\mu\beta}_{(FG)}(k) &= [-k^2 d_{\alpha\mu}]D^{\mu\beta}_{(FG)}(k) + \frac{k_\alpha k^\beta}{k^2}.
\end{aligned} \tag{3.25}$$

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Then we follow the prescription explained in Sect.2 and we obtain the pinch contributions to the gluon self-energy. Since the gluon propagator in FG does not have a longitudinal $k^\mu k^\nu$ term, the only contribution is coming from the vertex diagram of the second kind with the three-gluon vertex Γ^P (and its mirror graph) [17], and it is given by

$$\Pi^{\mu\nu}_{P(FG)}(k) = 2Ng^2 k^2 d^{\mu\nu} \int dp \frac{1}{p^2 q^2}. \tag{3.26}$$

Adding $\Pi^{\mu\nu}_{(FG)}$ and $\Pi^{\mu\nu}_{P(FG)}$, we find the sum coincides with the gauge independent $\widehat{\Pi}^{\mu\nu}(k)$ in Eq.(3.20). Thus we obtain the same Π_T in Eq.(3.15) and the same β_T in Eq.(3.16) while we work in FG.

3.3 The Coulomb Gauge

In fact it is rather anticipated that once we use PT for the one-loop calculation of the thermal β function, we obtain the ξ_Q-independent β_T in BFM which coincides with the result in FG. However, it is less trivial whether we may reach the same result for β_T when we calculate in noncovariant gauges such as the Coulomb gauge and the temporal axial gauge. We show in this and the following subsections that we indeed obtain the same β_T in the above two noncovariant gauges when we use PT.

Given a unit vector $n^\mu = (1, 0, 0, 0)$, the gluon propagator in the Coulomb gauge (CG), $iD^{\mu\nu}_{ab(CG)} = i\delta_{ab}D^{\mu\nu}_{(CG)}$, and its inverse are expressed as

$$D^{\mu\nu}_{(CG)}(k) = -\frac{1}{k^2}\left[g^{\mu\nu} + \left(1 - \xi_C \frac{k^2}{\vec{\mathbf{k}}^2}\right)\frac{k^\mu k^\nu}{\vec{\mathbf{k}}^2} - \frac{k_0}{\vec{\mathbf{k}}^2}(k^\mu n^\nu + n^\mu k^\nu)\right] \tag{3.27}$$

$$[D^{-1}_{(CG)}]^{\mu\nu}(k) = -k^2\left[g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right] + \frac{1}{\xi_C}\left[k^\mu k^\nu - k_0(k^\mu n^\nu + n^\mu k^\nu) + k_0^2 n^\mu n^\nu\right].$$

where ξ_C is the gauge parameter of CG. The three-gluon vertex is the same as in FG, that is, $\Gamma^{abc}_{\lambda\mu\nu}(p, k, q)$ in Eq.(3.22). It is noted that although in the limit $\xi_C = 0$, $D^{\mu\nu}_{(CG)}(k)$ reduces to the well-known form [24]

$$D^{00}_{(CG)} = \frac{1}{\vec{\mathbf{k}}^2}, \quad D^{0i}_{(CG)} = 0, \quad D^{ij}_{(CG)} = \frac{1}{k^2}\left(\delta^{ij} - \frac{k^i k^j}{\vec{\mathbf{k}}^2}\right), \tag{3.28}$$

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its inverse does not exist in this limit. In the framework of PT, we need to use the identities in Eq.(2.3), satisfied by the gluon propagator and its inverse, to extract from T_P the pinch contributions to the gluon self-energy. Thus in principle we must work with a non-zero ξ_C. Then at one-loop level there appear ξ_C-dependent terms in the gluon self-energy. We can show [23], however, that the one-loop pinch contributions are also ξ_C-dependent and these ξ_C-dependent parts exactly cancel against the ξ_C-dependent terms in the self-energy. For our purpose of calculating the thermal β function, therefore, it is enough to know the information on the ξ_C-independent part of both the self-energy and the pinch contributions. The ξ_C-independent part of the one-loop gluon self-energy was calculated in Ref.[25] using the gluon propagator in the $\xi_C = 0$ limit given in Eq.(3.28).

The transverse function Π_T is related to the self-energy as

$$\Pi_T = t_{ij}\Pi_{ij} = \frac{1}{2}\left[\Pi_{ii} - \frac{1}{\vec{\mathbf{k}}^2}k_i\Pi_{ij}k_j\right]. \tag{3.29}$$

Since $k_i\Pi_{ij}k_j = 0$ in the static limit $k_0 = 0$, we have $\Pi_T(T,\kappa) = \frac{1}{2}\Pi_{ii}(k_0 = 0,\kappa)$. The ξ_C-independent part of $\Pi_{ii}^{(CG)}(k)$ is given in Eq.(4.12) of Ref.[25] as

$$
\begin{aligned}
\Pi_{ii}^{(CG)}(k) &= \frac{N}{2}g^2\int dp\left\{\frac{8}{p^2} - \frac{6}{\vec{\mathbf{p}}^2}\right.\\
&\quad + \frac{1}{p^2q^2}\left[|\vec{\mathbf{p}} - \vec{\mathbf{q}}|^2(1+c^2) + 8(\vec{\mathbf{p}}^2 + \vec{\mathbf{q}}^2)(1-c^2)\right]\\
&\quad + \left.\left[2\frac{(k_0 - p_0)^2}{p^2\vec{\mathbf{q}}^2} + (p \leftrightarrow q)\right] + \frac{|\vec{\mathbf{p}} - \vec{\mathbf{q}}|^2}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2} + \frac{2\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2}\right\} \tag{3.30}
\end{aligned}
$$

Using the formulas given in Appendix B, we can calculate the static limit of Π_{ii}^{CG} and we obtain

$$
\begin{aligned}
\Pi_T^{(CG)}(T,\kappa) &= \frac{1}{2}\Pi_{ii}^{(CG)}(k_0 = 0,\kappa)\\
&= \frac{Ng^2}{4\pi^2}\int_0^\infty dp\, p\, n(p)\left\{-\frac{3}{2} + \frac{5\kappa^2}{4p^2} + \left[-\frac{p}{4\kappa} - \frac{11\kappa}{8p} + \frac{\kappa^3}{2p^3} + \frac{\kappa^5}{8p^5}\right]\ln\left|\frac{p+\kappa}{p-\kappa}\right|\right.\\
&\quad + \left.\left[\frac{2p}{\kappa} + \frac{5\kappa}{2p} - \frac{\kappa^3}{2p^3} - \frac{\kappa^5}{16p^5}\right]\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\right\} \tag{3.31}
\end{aligned}
$$

In the limit $\kappa \ll T$, this gives $\Pi_T^{(CG)}(T,\kappa) \approx Ng^2\kappa T\frac{9}{64} + \mathcal{O}(\kappa^2)$.

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Now we calculate the pinch contributions to the gluon self-energy in CG. Since the gluon propagator and its inverse satisfy the relations in Eq.(2.3), that is,

$$
\begin{aligned}
D_{\alpha\mu}^{(CG)}(k)[D_{(CG)}^{-1}]^{\mu\beta}(k) &= D_{\alpha\mu}^{(CG)}(k)[-k^2 d^{\mu\beta}] + \frac{k_\alpha}{\vec{\mathbf{k}}^2}(k_0 n^\beta - k^\beta) \\
D_{(CG)\alpha\mu}^{-1}(k)D_{(CG)}^{\mu\beta}(k) &= [-k^2 d_{\alpha\mu}]D_{(CG)}^{\mu\beta}(k) + (k_0 n_\alpha - k_\alpha)\frac{k^\beta}{\vec{\mathbf{k}}^2} \,, \qquad (3.32)
\end{aligned}
$$

again we can follow the prescription explained in Sec.2 to extract the one-loop pinch contributions. The individual contributions in CG from the vertex diagrams of the first kind, of the second kind and the box diagrams are presented in Appendix A. In total we obtain for the ξ_C-independent part

$$
\begin{aligned}
\Pi_{P(CG)}^{\mu\nu}(k) &= Ng^2 k^2 d^{\mu\nu} \int dp \frac{1}{p^2 q^2 \vec{\mathbf{p}}^2}(k^2 - q^2 - 4\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}) \\
&+ \frac{N}{2}g^2 k^2 d^{\mu\alpha} d^{\nu\beta} \int dp \frac{1}{p^2 q^2 \vec{\mathbf{p}}^2 \vec{\mathbf{q}}^2} \Big\{ p_\alpha p_\beta(k^2 - 4\vec{\mathbf{q}}^2 + 2\vec{\mathbf{k}}^2) \\
&\quad + (p_\alpha n_\beta + n_\alpha p_\beta)[p^2 q_0 - q^2 p_0 - 2\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}(p_0 - q_0)] + n_\alpha n_\beta 4 p_0 q_0 (pq) \Big\} \\
&+ \frac{N}{2}g^2 \Big[d^{\mu\alpha} \int dp \Big\{ p_\alpha k^\nu \Big[\frac{1}{q^2\vec{\mathbf{p}}^2} - \frac{1}{p^2\vec{\mathbf{q}}^2} + (\frac{1}{q^2} - \frac{1}{p^2})\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2} \Big] \\
&\quad + n_\alpha k^\nu \Big[-\frac{q_0}{p^2\vec{\mathbf{q}}^2} - \frac{p_0}{q^2\vec{\mathbf{p}}^2} + (\frac{q_0}{q^2} + \frac{p_0}{p^2})\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2} \Big] \Big\} + (\mu \leftrightarrow \nu) \Big]. \quad (3.33)
\end{aligned}
$$

It was pointed out [25] that the gluon self-energy caluculated in CG does not satisfy the transversality relation, i.e., $\Pi_{(CG)}^{\mu\nu}k_\nu \neq 0$. Now we can see that the pinch contribution $\Pi_{P(CG)}^{\mu\nu}$ does not satisfy the transversality relation either. In fact we get

$$
\begin{aligned}
\Pi_{P(CG)}^{\mu\nu}k_\nu &= \frac{N}{2}g^2 k^2 d^{\mu\alpha} \int dp \Big\{ p_\alpha \Big[\frac{1}{q^2\vec{\mathbf{p}}^2} - \frac{1}{p^2\vec{\mathbf{q}}^2} + (\frac{1}{q^2} - \frac{1}{p^2})\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2} \Big] \\
&\quad + n_\alpha \Big[-\frac{q_0}{p^2\vec{\mathbf{q}}^2} - \frac{p_0}{q^2\vec{\mathbf{p}}^2} + (\frac{q_0}{q^2} + \frac{p_0}{p^2})\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2} \Big] \Big\} \,. \qquad (3.34)
\end{aligned}
$$

It can be shown that this non-transverse part of pinch contribution exactly cancels against the non-transverse part of $\Pi_{(CG)}^{\mu\nu}k_\nu$ and that the sum of $\Pi_{(CG)}^{\mu\nu}$ and $\Pi_{P(CG)}^{\mu\nu}$ indeed satisfies the transversality relation [23].

The function $\Pi_T^{P(CG)}(T,\kappa)$ is obtained by applying the projection operator t_{ij} to the spatial part of $\Pi_{P(CG)}^{ij}(k_0 = 0, \kappa)$. Since $d^{i\alpha}n_\alpha = 0$ in the static limit and

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$t_{ij}k^j = 0$, the terms proportional to $(p_\alpha n_\beta + n_\alpha p_\beta)$, $n_\alpha n_\beta$, $d^{\mu\alpha}p_\alpha k^\nu$ and $d^{\mu\alpha}n_\alpha k^\nu$ in Eq.(3.33) do not contribute to $\Pi_T^{P(CG)}$. The result is

$$
\begin{aligned}
\Pi_T^{P(CG)}(T,\kappa) &= -Ng^2\kappa^2 \int dp \Big\{ \frac{1}{q^2\vec{\mathbf{p}}^2} + \frac{2\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^2q^2\vec{\mathbf{p}}^2} \Big\} \\
&\quad - \frac{N}{4}g^2\kappa^2 \int dp \Big[1 - \frac{(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2}{\vec{\mathbf{k}}^2\vec{\mathbf{p}}^2} \Big] \Big\{ \frac{\vec{\mathbf{k}}^2}{p^2q^2\vec{\mathbf{q}}^2} - \frac{4}{p^2q^2} \Big\} . \quad (3.35)
\end{aligned}
$$

Since the terms proportional to n and k do not contribute, the quickiest way to arrive at the above expression of $\Pi_T^{P(CG)}$ is that we consider the quark-quark scattering amplitude at one-loop order and discard the terms which are proportional to $\not{n}$ and $\not{k}$ from the beginning. In fact, this simplified method was used in Ref.[15] to obtain the pinch contribution Π_T^P in both the Coulomb gauge and the temporal axial gauge.

After the p_0 summation and the angular integration for $\Pi_T^{P(CG)}$ (see the formulas listed in Appendix B), we have

$$
\begin{aligned}
\Pi_T^{P(CG)}(T,\kappa) &= \frac{Ng^2}{4\pi^2} \int_0^\infty dp\, p\, n(p) \Big\{ -\frac{1}{2} - \frac{5\kappa^2}{4p^2} + \Big[\frac{p}{4\kappa} + \frac{11\kappa}{8p} - \frac{\kappa^3}{2p^3} - \frac{\kappa^5}{8p^5} \Big] \ln\Big| \frac{p+\kappa}{p-\kappa} \Big| \\
&\quad + \Big[\frac{\kappa}{p} + \frac{\kappa^3}{2p^3} + \frac{\kappa^5}{16p^5} \Big] \ln\Big| \frac{2p+\kappa}{2p-\kappa} \Big| \Big\}. \quad (3.36)
\end{aligned}
$$

In the limit $\kappa << T$, this gives $\Pi_T^{P(CG)}(T,\kappa) \approx Ng^2\kappa T\frac{19}{64} + \mathcal{O}(\kappa^2)$. Adding the two contributions, $\Pi_T^{(CG)}$ and $\Pi_T^{P(CG)}$, we find that the sum is equal to Π_T in Eq.(3.14) and, thus, we obtain the same β_T in Eq.(3.16).

3.4   The Temporal Axial Gauge

The gluon propagator in the temporal axial gauge (TAG), $iD_{ab(TAG)}^{\mu\nu} = i\delta_{ab}D_{(TAG)}^{\mu\nu}$, and its inverse are defined by

$$
D_{(TAG)}^{\mu\nu}(k) = -\frac{1}{k^2}\Big[g^{\mu\nu} + (1+\xi_A k^2)\frac{k^\mu k^\nu}{k_0^2} - \frac{1}{k_0}(k^\mu n^\nu + n^\mu k^\nu) \Big] \quad (3.37)
$$

$$
[D_{(TAG)}^{-1}]^{\mu\nu}(k) = -k^2\Big(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \Big) - \frac{1}{\xi_A}n^\mu n^\nu \quad (3.38)
$$

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where $n^\mu = (1, 0, 0, 0)$. It is noted that the gauge parameter ξ_A in TAG has a dimension of mass^{-2}. They satisfy the relations in Eq.(2.3):

$$D_{\alpha\mu}^{(TAG)}(k)[D_{(TAG)}^{-1}]^{\mu\beta}(k) = D_{\alpha\mu}^{(TAG)}(k)[-k^2 d^{\mu\beta}] + k_\alpha \left(\frac{n^\beta}{k_0} - \frac{k^\beta}{k^2} \right)$$

$$D_{(TAG)\alpha\mu}^{-1}(k)D_{(TAG)}^{\mu\beta}(k) = [-k^2 d_{\alpha\mu}]D_{(TAG)}^{\mu\beta}(k) + \left(\frac{n_\alpha}{k_0} - \frac{k_\alpha}{k^2} \right)k^\beta \qquad (3.39)$$

The three-gluon vertex is given by $\Gamma_{\lambda\mu\nu}^{abc}(p, k, q)$ in Eq.(3.22).

In the limit $\xi_A = 0$ the inverse of the gluon propagator does not exist. So in the framework of PT we must work with a non-zero ξ_A. Then ξ_A-dependent terms appear from both the one-loop gluon self-energy and the corresponding pinch contributions. But again they cancel each other [23]. So we only concern about the ξ_A-independent parts of the one-loop gluon self-energy and pinch contributions. It is noted that although the ghost field decouples in the limit $\xi_A = 0$, for a non-zero ξ_A the ghost should be taken into account and at one-loop level it contributes to the ξ_A-independent part of $\Pi_{00}^{(TAG)}$ [25][23].

Using the TAG propagator with $\xi_A = 0$, the ξ_A-independent part of $\Pi_{ii}^{(TAG)}$ at one-loop was calculated in Ref.[25]. The static limit of $\Pi_{ii}^{(TAG)}$ for $\kappa << T$ was given in Eq.(4.44) of Ref. [25] as

$$\Pi_{ii}^{(TAG)}(k_0 = 0, \kappa) \approx Ng^2\kappa T\frac{5}{8} + \mathcal{O}(\kappa^2) , \qquad (3.40)$$

from which we obtain

$$\Pi_T^{(TAG)}(T, \kappa) = \frac{1}{2}\Pi_{ii}^{(TAG)}(0, \kappa) \approx Ng^2\kappa T\frac{5}{16} + \mathcal{O}(\kappa^2). \qquad (3.41)$$

In Ref.[15] the pinch contribution to $\Pi_T(T, \kappa)$ in TAG was calculated and the result was for $\kappa << T$

$$\Pi_T^{P(TAG)}(T, \kappa) \approx Ng^2\kappa T\frac{1}{8} + \mathcal{O}(\kappa^2). \qquad (3.42)$$

Again the sum of $\Pi_T^{(TAG)}$ and $\Pi_T^{P(TAG)}$ coincides with Π_T in Eq.(3.15) and yields the same β_T in Eq.(3.16).

In the evaluation of $\Pi_{ii}^{(TAG)}(k_0 = 0, \kappa)$ and $\Pi_T^{P(TAG)}(T, \kappa)$, there appear $\vec{k}^2/\vec{p}^2$ singularities at the lower limit of the integration, due to the $1/p_0^2$ and $1/q_0^2$ terms

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coming from the TAG propagator. These singularities were circumvented [25][15] by the principal value prescription [26]. In the present paper we show instead that when the pinch contributions are added to $\Pi_T^{(TAG)}(T, \kappa)$ before the $p(= |\vec{p}|)$-integration, these $\vec{k}^2/\vec{p}^2$ singularities cancel and the limit $p \to 0$ becomes regular, so that we can evaluate the sum of $\Pi_T^{(TAG)}$ and $\Pi_T^{P(TAG)}$ without recourse to the principal value prescription. In fact Eq.(4.43) for $\Pi_{ii}^{(TAG)}(0, \kappa)$ in Ref.[25] will be rewritten as

$$
\begin{aligned}
\Pi_{ii}^{(TAG)}(0, \kappa) &= \frac{Ng^2}{2\pi^2} \int_0^\infty dp \, p \, n(p) \\
&\times \left[-2 + \frac{\kappa^2}{p^2} + \frac{\kappa^4}{4p^4} + \left(\frac{2p}{\kappa} + \frac{5\kappa}{2p} - \frac{\kappa^3}{2p^3} - \frac{\kappa^5}{16p^5} \right) \ln\left| \frac{2p + \kappa}{2p - \kappa} \right| \right]
\end{aligned}
\tag{3.43}
$$

if we do not apply the principal value prescription. We see that the integrand (the terms in $[\cdots]$) will behave as $-4\kappa^2/3p^2$ for small p.

Since the TAG propagator and its inverse satisfy the relation of Eq.(2.3), we can follow the same procedure as before and we obtain for the ξ_A-independent part of the pinch contribution to the gluon self-energy in TAG,

$$
\begin{aligned}
\Pi_{P(TAG)}^{\mu\nu} &= Ng^2 k^2 d^{\mu\nu} \int dp \frac{1}{p^2 q^2 p_0^2} (k^2 + 2p^2 - q^2 - 4\vec{k} \cdot \vec{p}) \\
&+ \frac{N}{2} g^2 k^2 d^{\mu\alpha} d^{\nu\beta} \int dp \frac{1}{p^2 q^2 p_0^2 q_0^2} \Big\{ p_\alpha p_\beta (k^2 - 4q_0^2 + 2\vec{k}^2) \\
&\quad + (p_\alpha n_\beta + n_\alpha p_\beta)[-p^2 q_0 + q^2 p_0 - 2\vec{p} \cdot \vec{q}(p_0 - q_0)] + n_\alpha n_\beta 4 p_0 q_0 (pq) \Big\}
\end{aligned}
\tag{3.44}
$$

Then in the static limit, $\Pi_T^{P(TAG)}$ is expressed as

$$
\begin{aligned}
\Pi_T^{P(TAG)}(T, \kappa) &= -Ng^2 \kappa^2 \int dp \Big\{ \frac{\vec{k}^2 + 4\vec{k} \cdot \vec{p}}{p^2 q^2 p_0^2} + \frac{1}{p^2 p_0^2} - \frac{2}{q^2 p_0^2} \Big\} \\
&- \frac{N}{4} g^2 \kappa^2 \int dp \Big[\vec{p}^2 - \frac{(\vec{k} \cdot \vec{p})^2}{\vec{k}^2} \Big] \Big\{ \frac{\vec{k}^2}{p^2 q^2 p_0^2 q_0^2} - \frac{4}{p^2 q^2 p_0^2} \Big\}. \tag{3.45}
\end{aligned}
$$

where the terms proportional to $(p_\alpha n_\beta + n_\alpha p_\beta)$ and $n_\alpha n_\beta$ in $\Pi_{P(TAG)}^{\mu\nu}$ do not contribute to $\Pi_T^{P(TAG)}$. After the p_0-summation and the angular integration, $\Pi_T^{P(TAG)}(T, \kappa)$ is

rewritten as

$$
\begin{aligned}
\Pi_T^{P(TAG)}(0, \kappa) &= \frac{Ng^2}{4\pi^2} \int_0^\infty dp \; p \; n(p) \\
&\quad \times \left[-\frac{\kappa^2}{p^2} - \frac{\kappa^4}{4p^4} + \left(\frac{\kappa}{p} + \frac{\kappa^3}{2p^3} + \frac{\kappa^5}{16p^5} \right) \ln\left| \frac{2p + \kappa}{2p - \kappa} \right| \right] ,
\end{aligned}
\tag{3.46}
$$

where we have used formulas given in Appendix B. Note that the integrand behaves as $4\kappa^2/3p^2$ for small p. When $\Pi_T^{(TAG)}$ and $\Pi_T^{P(TAG)}$ are combined (remember $\Pi_T^{(TAG)} = \frac{1}{2}\Pi_{ii}^{(TAG)}(0, \kappa)$), the κ^2/p^2 singularities cancel and the integrand becomes regular as $p \to 0$. Indeed we find the sum of $\Pi_T^{(TAG)}$ and $\Pi_T^{P(TAG)}$ coincides with Π_T in Eq.(3.14).

4 Summary and Discussion

The calculation of the thermal β function β_T was performed in four different gauges, that is, in BFM with an arbitrary gauge parameter, in FG, in CG, and in TAG. When the pinch contributions were taken care of, the same result $\beta_T = g^3 N \frac{7}{32} \frac{T}{\kappa}$ was obtained at one-loop order in all four cases.

However, this is not the end of the story. Elmfors and Kobes pointed out [11] that the leading contribution to β_T, which gives a term T/κ, does not come from the hard part of the loop integral, responsible for a T^2/κ^2 term, but from soft loop integral. Hence they emphasized that it is not consistent to stop the calculation at one-loop order for soft internal momenta and that the resummed propagator and vertices [27] must be used to get the complete leading contribution. The need for resummation is urged also by the following observation: The fact that we have obtained the same β_T at one-loop level in four different gauges implies that the effective gluon self-energy $\widehat{\Pi}^{\mu\nu}$ in Eq.(3.20), constructed in BFM or in FG with recourse to PT, is gauge-fixing independent and universal. Provided that we use $\widehat{\Pi}^{\mu\nu}$ for calculation of the gluon damping rate γ at zero momentum, we would obtain $\gamma = -11Ng^2T/(24\pi)$ [28], a negative damping rate, which is not acceptable today [27][29].

Since the corrections to the bare propagator and vertices, which come from the hard thermal loops, are gauge-independent and satisfy simple Ward identities [27], it is well-expected that we will obtain the gauge-independent thermal β function even when we use the resummed propagator and vertices in the framework of PT. Study along this direction is under way.

A Pinch contributions to gluon self-energy

In this Appendix we give the one-loop pinch contributions to the gluon self-energy from the vertex diagrams of the first kind (V_1), the vertex diagrams of the second kind (V_2), and box diagrams (Box), separately, calculated in four different gauges: (i) The background field method with an arbitrary ξ_Q; (ii) The Feynman gauge; (iii) The Coulomb gauge; (iv) The temporal axial gauge. The results in the cases of the background field method [13] and the Feynman gauge [17] are already known, but they are listed again for completeness. The expressions are in the imaginary time formalism and thus

$$\int dp = \int \frac{d^3 p}{(2\pi)^3} \, T \sum_n \tag{A.1}$$

where the summation goes over $p_0 = 2\pi i n T$. They are transformed into the ones in the Minkowski space by the replacement $\int dp \Rightarrow -i \int d^4 p/(2\pi)^4$.

A.1 *The background field method with an arbitrary ξ_Q*

$$\Pi^{\mu\nu(V_1)}_{P(BFM)} = N g^2 (1 - \xi_Q) k^2 d^{\mu\nu} \int dp \frac{1}{p^4} \tag{A.2}$$

$$\begin{aligned}
\Pi^{\mu\nu(V_2)}_{P(BFM)} = {} & N g^2 (1 - \xi_Q) k^2 d^{\mu\nu} \int dp \left[-\frac{1}{p^2 q^2} - \frac{4kp}{p^4 q^2} \right] \\
& + \frac{N}{2} g^2 (1 - \xi_Q)^2 k^4 d^{\mu\alpha} d^{\nu\beta} \int dp \frac{-2 p_\alpha p_\beta}{p^4 q^4}
\end{aligned} \tag{A.3}$$

$$\begin{aligned}
\Pi^{\mu\nu(Box)}_{P(BFM)} = {} & N g^2 (1 - \xi_Q) k^4 d^{\mu\nu} \int dp \frac{-1}{p^4 q^2} \\
& + \frac{N}{2} g^2 (1 - \xi_Q)^2 k^4 d^{\mu\alpha} d^{\nu\beta} \int dp \frac{p_\alpha p_\beta}{p^4 q^4}
\end{aligned} \tag{A.4}$$

A.2 *The Feynman gauge*

There are no contributions from the box and the vertex diagrams of the first kind. Thus we have $\Pi^{\mu\nu(V_1)}_{P(FG)} = \Pi^{\mu\nu(Box)}_{P(FG)} = 0$. The only contribution to the pinch part

comes from the vertex diagrams of the second:

$$\Pi^{\mu\nu(V_2)}_{P(FG)} = 2Ng^2k^2d^{\mu\nu}\int dp\frac{1}{p^2q^2}. \tag{A.5}$$

A.3   The Coulomb gauge

Only the ξ_C-independent parts of the pinch contributions are listed.

$$\Pi^{\mu\nu(V_1)}_{P(CG)} = Ng^2k^2d^{\mu\nu}\int dp\frac{-1}{p^2\vec{\mathbf{p}}^2} \tag{A.6}$$

$$
\begin{aligned}
\Pi^{\mu\nu(V_2)}_{P(CG)} = {} & Ng^2k^2d^{\mu\nu}\int dp\frac{-4\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^2q^2\vec{\mathbf{p}}^2} \\
& + Ng^2k^2d^{\mu\alpha}d^{\nu\beta}\int dp\frac{1}{p^2q^2\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2}\Big\{p_\alpha p_\beta(\vec{\mathbf{k}}^2-2\vec{\mathbf{q}}^2)+n_\alpha n_\beta p_0q_0(k^2+2pq) \\
& \qquad\qquad + (p_\alpha n_\beta+n_\alpha p_\beta)\frac{1}{2}[p_0p^2-q_0q^2+2(p_0-q_0)(p_0q_0-2\vec{\mathbf{p}}\cdot\vec{\mathbf{q}})]\Big\} \\
& + \frac{N}{2}g^2\Big[d^{\mu\alpha}\int dp\Big\{p_\alpha k^\nu\Big[\frac{1}{q^2\vec{\mathbf{p}}^2}-\frac{1}{p^2\vec{\mathbf{q}}^2}+(\frac{1}{q^2}-\frac{1}{p^2})\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2}\Big] \\
& \qquad\qquad + n_\alpha k^\nu\Big[-\frac{q_0}{p^2\vec{\mathbf{q}}^2}-\frac{p_0}{q^2\vec{\mathbf{p}}^2}+(\frac{q_0}{q^2}+\frac{p_0}{p^2})\frac{\vec{\mathbf{p}}\cdot\vec{\mathbf{q}}}{\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2}\Big]\Big\}+(\mu\leftrightarrow\nu)\Big] \tag{A.7}
\end{aligned}
$$

$$
\begin{aligned}
\Pi^{\mu\nu(Box)}_{P(CG)} = {} & Ng^2k^4d^{\mu\nu}\int dp\frac{1}{p^2q^2\vec{\mathbf{p}}^2} \\
& + \frac{N}{2}g^2k^4d^{\mu\alpha}d^{\mu\beta}\int dp\frac{1}{p^2q^2\vec{\mathbf{p}}^2\vec{\mathbf{q}}^2}[p_\alpha p_\beta+(p_\alpha n_\beta+n_\alpha p_\beta)(q_0-p_0) \\
& \qquad\qquad\qquad\qquad -2n_\alpha n_\beta p_0q_0] \tag{A.8}
\end{aligned}
$$

A.4   The temporal axial gauge

Only the ξ_A-independent parts of the pinch contributions are listed.

$$\Pi^{\mu\nu(V_1)}_{P(TAG)} = Ng^2k^2d^{\mu\nu}\int dp\frac{-1}{p^2p_0^2} \tag{A.9}$$

$$
\begin{aligned}
\Pi^{\mu\nu(V_2)}_{P(TAG)} = {} & Ng^2k^2d^{\mu\nu}\int dp\frac{1}{p^2q^2}\Big\{\frac{2p^2}{p_0^2}-\frac{4\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p_0^2}\Big\} \\
& + Ng^2k^2d^{\mu\alpha}d^{\nu\beta}\int dp\frac{1}{p^2q^2p_0^2q_0^2}\Big\{p_\alpha p_\beta[\vec{\mathbf{k}}^2-2q_0^2]+n_\alpha n_\beta p_0q_0(k^2+2pq)
\end{aligned}
$$

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$$+(p_\alpha n_\beta + n_\alpha p_\beta)[\frac{1}{2}(p_0 p^2 - q_0 q^2) - p_0(\vec{\mathbf{q}}^2 + 2\vec{\mathbf{p}} \cdot \vec{\mathbf{q}}) + q_0(\vec{\mathbf{p}}^2 + 2\vec{\mathbf{p}} \cdot \vec{\mathbf{q}})]\Big\}$$

$$\text{(A.10)}$$

$$\Pi^{\mu\nu(Box)}_{P(TAG)} = Ng^2 k^4 d^{\mu\nu} \int dp \frac{1}{p^2 q^2 p_0^2}$$
$$+ \frac{N}{2} g^2 k^4 d^{\mu\alpha} d^{\nu\beta} \int dp \frac{p_\alpha p_\beta + (p_\alpha n_\beta + n_\alpha p_\beta)(q_0 - p_0) - 2n_\alpha n_\beta p_0 q_0}{p^2 q^2 p_0^2 q_0^2} \text{(A.11)}$$

B   Thermal one-loop integrals

We list the thermal one-loop integrals in the static limit $k_0 = 0$ which are used in this paper. We only give the matter part. Due to the constraint $k + p + q = 0$ there holds a relation

$$\int dp f(p, q) = \int dp f(q, p). \qquad \text{(B.1)}$$

It is understood that in the r.h.s. of the expressions below, $p \equiv |\vec{\mathbf{p}}|$, $\kappa \equiv |\vec{\mathbf{k}}|$ and $n(p) = 1/[\exp(p/T) - 1]$.

$$\int dp \frac{p_0^2}{p^2 q^2} = \frac{1}{4\pi^2 \kappa} \int_0^\infty dp \ p^2 \ n(p) \ \ln\left|\frac{2p + \kappa}{2p - \kappa}\right| \qquad \text{(B.2)}$$

$$\frac{1}{\vec{\mathbf{k}}^2} \int dp \frac{(\vec{\mathbf{k}} \cdot \vec{\mathbf{p}})^2}{p^2 q^2} = \frac{1}{8\pi^2} \int_0^\infty dp \ p \ n(p) \left\{ 2 + \frac{\kappa}{2p} \ \ln\left|\frac{2p + \kappa}{2p - \kappa}\right| \right\} \qquad \text{(B.3)}$$

$$\vec{\mathbf{k}}^2 \int dp \frac{1}{p^2 q^2} = \frac{\kappa}{4\pi^2} \int_0^\infty dp \ n(p) \ \ln\left|\frac{2p + \kappa}{2p - \kappa}\right| \qquad \text{(B.4)}$$

$$\vec{\mathbf{k}}^2 \int dp \frac{\vec{\mathbf{k}} \cdot \vec{\mathbf{p}}}{p^4 q^2} = \frac{\kappa}{8\pi^2} \int_0^\infty dp \ n(p) \left\{ -\frac{4p\kappa}{(2p + \kappa)(2p - \kappa)} + \ln\left|\frac{2p + \kappa}{2p - \kappa}\right| \right\} \qquad \text{(B.5)}$$

$$\int dp \frac{\vec{\mathbf{k}}^2 \vec{\mathbf{p}}^2 - (\vec{\mathbf{k}} \cdot \vec{\mathbf{p}})^2}{p^4 q^2} = -\frac{1}{2}\vec{\mathbf{k}}^2 \int dp \frac{1}{p^2 q^2} \qquad \text{(B.6)}$$

$$\vec{\mathbf{k}}^2 \int dp \frac{\vec{\mathbf{k}}^2 \vec{\mathbf{p}}^2 - (\vec{\mathbf{k}} \cdot \vec{\mathbf{p}})^2}{p^4 q^4} = \frac{\kappa}{4\pi^2} \int_0^\infty dp \ p \ n(p) \left\{ -\frac{2\kappa^2}{(2p + \kappa)(2p - \kappa)} + \frac{\kappa}{2p} \ \ln\left|\frac{2p + \kappa}{2p - \kappa}\right| \right\} \qquad \text{(B.7)}$$

$$\int dp \frac{1}{p^2} = -\frac{1}{2\pi^2} \int_0^\infty dp\; p\; n(p) = -\frac{1}{12}T^2 \tag{B.8}$$

$$\int dp \frac{1}{\vec{\mathbf{p}}^2} = 0 \qquad (\text{for matter part}) \tag{B.9}$$

$$\int dp \frac{p_0^2}{p^2\vec{\mathbf{q}}^2} = -\frac{1}{4\pi^2\kappa} \int_0^\infty dp\; p^2\; n(p)\; \ln\left|\frac{p+\kappa}{p-\kappa}\right| \tag{B.10}$$

$$\int dp \frac{1}{p^2 q^2}\Big[|\vec{\mathbf{p}} - \vec{\mathbf{q}}|^2(1+c^2) + 8(\vec{\mathbf{p}}^2 + \vec{\mathbf{q}}^2)(1-c^2)\Big]$$
$$= \frac{1}{2\pi^2}\int_0^\infty dp\; p\; n(p)\Bigg\{5 + \frac{5}{2}\frac{\kappa^2}{p^2} + \frac{(p^2-\kappa^2)^2(6p^2+\kappa^2)}{4p^5\kappa}\ln\left|\frac{p+\kappa}{p-\kappa}\right|$$
$$+ \frac{32p^6 + 40p^4\kappa^2 - 8p^2\kappa^4 - \kappa^6}{8p^5\kappa}\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\Bigg\} \tag{B.11}$$

$$\vec{\mathbf{k}}^2 \int dp \frac{1}{p^2\vec{\mathbf{q}}^2} = -\frac{\kappa}{4\pi^2}\int_0^\infty dp\; n(p)\; \ln\left|\frac{p+\kappa}{p-\kappa}\right| \tag{B.12}$$

$$\vec{\mathbf{k}}^2 \int dp \frac{\vec{\mathbf{k}}\cdot\vec{\mathbf{p}}}{p^2 q^2\vec{\mathbf{p}}^2} = \frac{\kappa}{4\pi^2}\int_0^\infty dp\; n(p)\Bigg\{\frac{\kappa}{p} + \frac{\kappa^2-p^2}{2p^2}\ln\left|\frac{p+\kappa}{p-\kappa}\right| - \frac{\kappa^2}{2p^2}\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\Bigg\} \tag{B.13}$$

$$\vec{\mathbf{k}}^2 \int dp\Big[1 - \frac{(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2}{\vec{\mathbf{k}}^2\vec{\mathbf{p}}^2}\Big]\frac{1}{p^2 q^2} = \frac{\kappa}{4\pi^2}\int_0^\infty dp\; n(p)\Bigg\{-\frac{p}{2\kappa} + \frac{\kappa}{2p}$$
$$- \Big[1 - \frac{(\kappa^2+p^2)^2}{4\kappa^2 p^2}\Big]\ln\left|\frac{p+\kappa}{p-\kappa}\right| + \Big(1 - \frac{\kappa^2}{4p^2}\Big)\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\Bigg\} \tag{B.14}$$

$$\vec{\mathbf{k}}^4 \int dp\Big[1 - \frac{(\vec{\mathbf{k}}\cdot\vec{\mathbf{p}})^2}{\vec{\mathbf{k}}^2\vec{\mathbf{p}}^2}\Big]\frac{1}{p^2 q^2\vec{\mathbf{q}}^2} = \frac{\kappa}{4\pi^2}\int_0^\infty dp\; n(p)\Bigg\{-\frac{\kappa}{p}$$
$$- \Big[1 - \frac{(\kappa^2+p^2)^2}{4\kappa^2 p^2}\Big]\frac{2\kappa^2}{p^2}\ln\left|\frac{p+\kappa}{p-\kappa}\right| + \Big(1 - \frac{\kappa^2}{4p^2}\Big)\frac{\kappa^2}{p^2}\ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\Bigg\} \tag{B.15}$$

$$\vec{k}^2 \int dp \frac{\vec{k}^2 + 4\vec{k}\cdot\vec{p}}{p^2 q^2 p_0^2} \;=\; \frac{1}{4\pi^2} \int_0^\infty dp\, p\, n(p)\left(-\frac{\kappa^3}{p^3}\right) \ln\left|\frac{2p+\kappa}{2p-\kappa}\right| \qquad \text{(B.16)}$$

$$\vec{k}^2 \int dp \frac{1}{q^2 p_0^2} \;=\; \frac{1}{4\pi^2} \int_0^\infty dp\, p\, n(p)\left(-2\frac{\kappa^2}{p^2}\right) \qquad \text{(B.17)}$$

$$\vec{k}^2 \int dp \frac{1}{p^2 p_0^2} \;=\; \frac{1}{4\pi^2} \int_0^\infty dp\, p\, n(p)\left(-2\frac{\kappa^2}{p^2}\right) \qquad \text{(B.18)}$$

$$\vec{k}^4 \int dp \left[\vec{p}^2 - \frac{(\vec{k}\cdot\vec{p})^2}{\vec{k}^2}\right] \frac{1}{p^2 q^2 p_0^2 q_0^2}$$
$$= \frac{1}{4\pi^2} \int_0^\infty dp\, p\, n(p)\left\{\frac{\kappa^4}{p^4} + \frac{\kappa^3(4p^2 - \kappa^2)}{4p^5} \ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\right\} \qquad \text{(B.19)}$$

$$\vec{k}^2 \int dp \left[\vec{p}^2 - \frac{(\vec{k}\cdot\vec{p})^2}{\vec{k}^2}\right] \frac{1}{p^2 q^2 p_0^2}$$
$$= \frac{1}{4\pi^2} \int_0^\infty dp\, p\, n(p)\left\{\frac{\kappa^2}{p^2} + \frac{\kappa(4p^2 - \kappa^2)}{4p^3} \ln\left|\frac{2p+\kappa}{2p-\kappa}\right|\right\} \qquad \text{(B.20)}$$

For the $\kappa << T$ expansion we use the following formulas [25]:

$$\frac{1}{2} \ln\left|\frac{1+x}{1-x}\right| \;=\; \sum_{r=1} \frac{1}{2r-1} x^{2r-1} \qquad (x < 1) \qquad \text{(B.21)}$$

$$= \sum_{r=1} \frac{1}{2r-1} x^{1-2r} \qquad (x > 1) \qquad \text{(B.22)}$$

$$\int_0^1 dx \frac{x^s}{e^{yx} - 1} \;=\; \frac{1}{sy} + \cdots \qquad \text{(B.23)}$$

$$\int_1^\infty dx \frac{1}{x^s(e^{yx} - 1)} \;=\; \frac{1}{sy} + \cdots \qquad \text{(B.24)}$$

$$= \frac{1}{y} + \frac{1}{2} \ln y + \cdots, \qquad \text{if } s = 1 \qquad \text{(B.25)}$$

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Figure caption

Fig.1

The self-energy diagram for the quark-quark scattering.

Fig.2

(a) The vertex diagrams of the first kind for the quark-quark scattering. (b) Their pinch contribution.

Fig.3

(a) The vertex diagram of the second kind for the quark-quark scattering. (b) Its pinch contribution.

Fig.4

(a) The box diagrams for the quark-quark scattering. (b) Their pinch contribution.